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THE CONCEPTION AND DEMONSTRATION OF ELECTRON WAVES

BY

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A REVIEW OF
THE GROWTH OF IDEAS REGARDING THE ELECTRON FROM THEIR INCEPTION LESS THAN ONE HUNDRED YEARS AGO TO THE PRESENT DAY

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The Conception and Demonstration of Electron Waves

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An attempt is made in this article to trace the growth of our ideas regarding the electron from their inception less than a hundred years ago to the present day. The discussion begins with a consideration of the vague and tentative deductions concerning an ultimate electrical charge which became possible when Faraday revealed the laws of electrolytic conduction; it touches upon the clarification of the conception of the electron as a charged particle capable of independent existence and subject to the laws of classical electrodynamics which was effected at the close of the last century and the beginning of the present one by the researches of J. J. Thomson and others; it indicates the difficulties in which this conception became involved, and the attempts made by Planck, Bohr and others to extricate it from them. The latter part of the paper is devoted to the amplified conception of the electron which has been developed during the last decade—a conception in which electrons are recognized as having, in different circumstances, the properties of both waves and particles.

Introduction

It is my purpose in this report to describe a few experiments, typical of several hundred now recorded, in which streams of electrons exhibit the properties of beams of waves. It seems desirable, however, to begin by briefly reviewing various steps in the development of our conception of the electron before about the year 1925. It is against this background only that the phenomena revealed by the experiments to be described appear in true relief.

The idea that electric charge is granular was not new at the time of the first International Electrical Congress in 1881. Faraday had determined and announced the laws of electrolytic conduction fifty years earlier, and it was recognized, by some at least, that these laws suggested the existence of an elementary charge or atom of electricity. An estimate of the magnitude of this natural and presumably ultimate unit of charge had indeed been made a few years prior to the Congress by the Irish physicist Stoney from such data as were then available. The word "electron" to designate the hypothetical atom of electricity was not, however, introduced until the year 1891. The concept of the electron gained rapidly in sharpness in the decade next following—the last of the century—not so much indeed from the introduction of new ideas concerning it as from experimental evidence in support of ideas already held.

It was no new idea, for example, that neutral atoms contain positive and negative charge in equal amounts, and that the ions of an electrolyte are merely atoms or groups of atoms in which these charges are

unbalanced in one direction or the other by one or more electronic units. Yet this remained largely a speculation until the study of the conductivity imparted to gases by X-rays made all other views untenable. Ions of both signs are formed at a uniform rate within the body of a gas subjected to this then newly discovered radiation; their charges are ionic; they move through the gas by diffusion and under the influence of an impressed electric field; they disappear through recombination.

Other ideas now familiar were not entirely novel even in 1890; for instance, the idea that positive and negative electrons possess mass as well as charge—that those of one sign are more massive than the other—that within the atom the lighter revolve about the heavier ones. Weber, following on Ampere, had pictured a mechanism of this kind to explain the magnetic properties of materials. All these notions became much more plausible, however, when Lorentz showed (as he did in 1897) that the splitting and polarization of spectral lines by a magnetic field might be explained as the effect of the field upon the period of revolving particles such as Weber had assumed, and that from the magnitude of this so-called "Zeeman effect" one might actually calculate the ratio of the charge of the particle to its mass. The value so found was greater by a factor 2000, or thereabouts, than the similar ratio for hydrogen ions in electrolysis. If the charge of the particle were indeed the electronic charge, then the mass of the particle was about 1/2000 only of the mass of the hydrogen atom—a highly acceptable conclusion. The concept of the electron had gained much in definiteness, and so also had that of the atom.

But more illuminating still was the discovery made in the same year that the trajectories followed by cathode rays in traversing electric and magnetic fields are exactly those to be expected if these rays are streams of swiftly moving negatively charged particles with a charge to mass ratio amounting again, as in the foregoing instance, to about 1/2000 that of the hydrogen ion. There could be little doubt that this was the very particle inferred by Lorentz from the "Zeeman effect." Cathode rays were certainly streams of free negative electrons—unattached to atoms. This supremely important discovery was made by J. J. Thomson in England and by Wiechert in Germany.

The conception of the negative electron as a subatomic particle possessing mass as well as charge, capable of independent existence, and subject to the laws of classical electrodynamics now seemed clearly established. If various of the simple relationships were at this time sensed rather than demonstrated—such, for example, as the exact identity of the charge to mass ratios of the Zeeman effect particle and the

cathode particle—there was, nevertheless, full confidence that these relationships would be confirmed by more exact measurements. And this indeed proved to be true. The anticipated details of the picture as then blocked in have since been supplied by a series of precision experiments in which Millikan's measurement of the absolute magnitude of the electronic charge is preeminent.

The turn of the century was a time of high hope. The key had been found, it appeared, to an understanding of vast ranges of phenomena; given the electron, electrodynamics and sufficient mathematics, *all* electrical and magnetic phenomena must become explicable. It seemed not too daring even to have thoughts concerning the structure of the atom. But this, as it turned out, was mostly illusion; every success of the electron theory of this period was matched by an equally conspicuous failure. Metallic conductors were pictured as containing atmospheres of free electrons with the properties of a monatomic gas. The drift of this electronic gas under the influence of an impressed field constituted the electric current. The form of Ohm's law was neatly explained, but not so the direct proportionality between the resistivity of a pure metal and its absolute temperature. The thermionic emission of electrons could be explained, apparently, in all its details, but the distribution of energy in the black body spectrum could not. The explanation of the simple Zeeman effect was most gratifying and reassuring, but the simple numerical relationships among the frequencies of line spectra remained as baffling as ever—and this, in spite of the considerable success which Drude and others had achieved in explaining the optical properties of materials in terms of electrons elastically bound within atoms.

The impasse was finally breached by Planck who showed, in 1905, that the black body spectrum could be explained if one were willing to assume that materials contain electric oscillators which emit and absorb energy only in amounts proportional to their frequencies. The conception of the electron was unaltered—not even involved, perhaps—but an oscillator, which might be a vibrating electron, was conceived to behave in a manner contrary to electrodynamical principles. A success had been achieved at the cost of violence to classical ideas regarding the production of electromagnetic radiation.

The next assault—a brilliant tour de force by Bohr—achieved its first objectives at a stride, but at a sacrifice of electrodynamical principles greater even than Planck's. Bohr showed in 1911 that by combining the idea of a concentrated atom nucleus required by Rutherford's experiments on the scattering of alpha rays with the heterodox idea of Planck, and with new devices of his own invention, one could

conceive an atom model capable of yielding precisely the Rydberg constant and the complete spectrum of atomic hydrogen. The casualties included two properties previously allotted to the electron as a matter of course: the property of radiating energy during orbital motion, and the property of revolving about the nucleus in an orbit consonant with classical dynamics and determined by initial conditions which might be regarded as arbitrary. Bohr excluded from the infinity of such orbits all but a special series. The motion of the electron remained planetary, but all else was new and bizarre.

A great initial success had, however, been attained and hope of a thorough conquest of spectra ran high,—too high as it now appears, for beyond a few other quantitative successes, further achievements were qualitative to a greater or less extent and consequently less impressive. It turned out also that advancement in the elucidation of spectra could be made only at the cost of an ever increasing array of special rules and prohibitions—additional equipment of the same arbitrary nature as that of Bohr's original postulates. Out of this necessity appeared the one new idea regarding the electron which had emerged in twenty years—the idea advanced by Goudsmit and Uhlenbeck that the electron spins and possesses in consequence a magnetic moment.

It was recognized a decade ago by Bohr and others that the attack upon the atom, despite its propitious beginning, had in a considerable measure failed; and this because it had lacked, so to speak, a proper base of operation. It was felt that the many arbitrary rules and restrictions required to correlate the data of spectroscopy must flow in a natural and unforced way from fundamental mechanical principles as yet undiscovered. A system of mechanics was envisaged which would degenerate to classical mechanics for large scale phenomena, but which would present an entirely different aspect on the atomic scale, and be capable, of course, of explaining atom dynamics as revealed by spectra.

Attempts to discover these underlying principles led to the formulation by Heisenberg in 1925 of what is known as matrix-mechanics, and led L. de Broglie in the same year to put forward his ideas concerning a so-called wave-mechanics. These proposals are said to be statements in different forms of one and the same principle—so far, at any rate, as applications to atom dynamics are concerned. It is the wave-mechanics, however, which has appealed most strongly to the physicist, and it is with this only that I will here concern myself.

The basic idea of the wave-mechanics was supplied by a paradoxical situation which had existed for some years in the theory of optics. It

was well established experimentally that a beam of monochromatic light can impart to individual electrons in matter amounts of energy proportional to its frequency. The factor of proportionality between these quantities—the energy imparted to the electrons and the frequency of the light—is the same as that obtained from the black body spectrum for the factor of proportionality between the energy quantum of the Planck oscillator and its frequency. The relation between the energy ϵ imparted to the electron and the frequency ν of the light is expressed, that is, by the formula $\epsilon = h\nu$, where h is the so-called Planck constant. When one tries to visualize the mechanism back of this phenomenon, he is led inevitably to a corpuscular theory of light. No other view appears adequate to explain this central fact of photoelectricity and others related to it.

On the other hand, the phenomena of interference and diffraction disposed long ago, as is well known, of an earlier corpuscular theory of light in favor of the wave theory. The demands of these phenomena are as insistent today as every they were, so that the situation comes to this, that one class of optical phenomena indicates clearly that light is a corpuscular radiation, and another indicates no less clearly that it is a propagation of waves. It is hopeless to try explaining photoelectric phenomena in terms of nothing but waves, and it is equally hopeless trying to devise a purely corpuscular interpretation of interference and diffraction.

It was de Broglie's brilliant idea that a situation similar to this might exist in regard to electrons, that electron streams like beams of light might possess in different circumstances the properties both of wave trains and of particle streams. If this were true, and if the wave aspect alone were adequate to explain the behavior of electrons in atoms, then the unhappy state of affairs which existed in regard to the interpretation of spectroscopic data might be remedied.

The formula $\epsilon = h\nu$ expresses, as we have seen, a certain correlation between the corpuscular and the wave properties of light; if the light regarded as a beam of waves is of frequency ν, then when it is regarded as a stream of corpuscles, the corpuscles or photons are of energy $\epsilon = h\nu$. A second correlation follows at once from this one and from the relation which is known to exist between the transfer of energy and of momentum by a beam of light. This second correlation relates the momentum p of the photon to the wave-length λ in vacuo of the associated waves, and is expressed by the formula $p = h/\lambda$. Or if we write σ to represent wave number—the number of waves per cm.—then the two correlations are expressed by the symmetrical formulæ

$$\epsilon = h\nu,$$
$$p = h\sigma.$$

In developing his idea of a possible wave aspect of the electron, de Broglie was led to the conclusion (partly, it appears by intuition and partly by considerations based on relativistic mechanics) that if electrons possess wave properties, the correlation between their wave and corpuscular aspects will be expressed by these same two formulæ.

It would lead us too far afield to follow even cursorily the further development of de Broglie's idea toward its original objective of explaining the behavior of atoms as disclosed by their spectra. These interesting matters must be left with the mere statement that the mathematical researches of Schrödinger and others have led to a conception of the atom in which standing wave patterns replace the permitted electron orbits of the Bohr model, and from which it is possible to derive certain laws of spectra which previously could be given only as empirical rules.

The immediate object which de Broglie had in view in postulating a wave aspect of the electron has thus been attained, and its attainment argues strongly, of course, for the soundness of the underlying conception. It is not this spectroscopic evidence, however, which reveals most clearly the wave as a real and actual property of electrons, but the more direct and unequivocal evidence supplied by experiments described in following paragraphs in which streams of electrons are diffracted by crystals.

It was implicit in de Broglie's earliest writings regarding electron waves that a stream of electrons moving with uniform speed along parallel lines will exhibit in appropriate circumstances the properties of a beam of monochromatic waves. De Broglie's first step was, indeed, to associate a train of plane parallel waves with an electron moving with uniform speed along a straight line. It remained, however, for the young German physicist Elsasser to point out the logical conclusion to which these speculations lead, and to indicate the crucial experiment by which they might be tested: to wit, that a beam of electrons scattered by an appropriate grating should exhibit the phenomenon of diffraction, and that the appropriate grating for this purpose is a crystal, since the wave-lengths calculated from de Broglie's formula for electrons of moderate speeds are like those of X-rays, of the order of one Angstrom unit.

It is the demonstration of this phenomenon—the diffraction of electrons by crystals—which constitutes now, as has been intimated, the chief experimental evidence in support of de Broglie's conception, and it is with demonstrations of this kind that I am here primarily concerned. The first of these was made by Davisson and Germer, who showed in 1927 that beams of electrons are diffracted by a crystal of

nickel, and that the wave-lenghts λ deduced from the diffraction-patterns for beams of electrons of various speeds v agree with those calculated from de Broglie's formula $\lambda = h/p = h/mv$. A second and independent demonstration was made by G. P. Thomson, who showed later in the same year that beams of high speed electrons are diffracted on transmission through thin films of polycrystalline metal, and that electron wave-lengths computed from patterns so obtained verify the de Broglie relationship.

It will be well, before considering these earliest experiments in more detail, to present certain others, made more recently, of which the interpretations are more simple. The simplest experimental result which suggests that electrons should be regarded as waves rather than as particles, is perhaps the regular reflection of a beam of electrons from the face of a crystal. The experimental arrangement used in demonstrating this phenomenon is indicated on the left in Fig. 1.

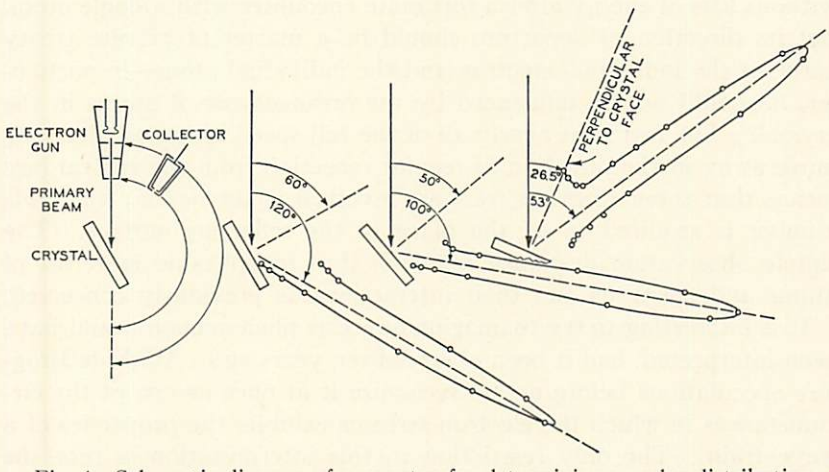


Fig. 1—Schematic diagram of apparatus for determining angular distribution of electrons scattered without loss of energy by a metallic crystal, and curves revealing specular reflection of 20-volt electrons.

Electrons emitted by a hot filament are accelerated and formed into a beam, and this beam is directed against the face of a crystal target at a known angle of incidence. The target in this case is of nickel and its face is parallel to one of the principal sets of atom planes of the crystal (111 planes). The surface is etched and presents to the incident beam a multitude of crystal facets parallel to the plane of the target face. Some of the electrons on striking these facets are scattered without loss of energy. The distribution of these full speed electrons in and near the plane of incidence is then determined by explorations with a

Faraday collector as indicated. The results of investigations of this kind, exhibited by polar diagrams in the same figure, show clearly that the incident beam is regularly reflected as if from the crystal facets.

The difficulty in explaining this result by the simple concept of electrons as particles is, that the surface of the crystal is much too coarse grained to serve as a reflector for particles as small as electrons; the diameter of the electron is of the order 10^{-13} cm., whereas, the diameters of atoms are of the order 10^{-8} cm.—greater by a factor 10^5—and this also is the order of the distance of least separation of atoms in the crystal face. It is hard to imagine how such a surface can appear smooth to the incident electrons. On the older views regarding interactions between electrons and atoms, the fate of an incident electron should be much the same as the fate of a comet plunging into a region densely packed with solar systems; the electron *might* emerge from the crystal without loss of energy after a fortunate encounter with a single atom, but its direction of departure should be a matter of private treaty between the individual electron and the individual atom—in particular, it should not be influenced by the arrangement of atoms in the crystal. The fact that nearly all of the full speed scattered electrons move away in the direction of regular reflection from the crystal face means that three atoms at least are involved in the action, since this number is required to fix the plane of the reflecting surface. The simple observation described above is thus inexplicable in terms of atoms and electrons and their interactions as previously conceived.

It is interesting to try to imagine how this phenomenon would have been interpreted, had it been observed ten years ago. With de Broglie's speculations before us, we recognize it at once as one of the circumstances in which the electron streams exhibits the properties of a wave train. The only restriction to this interpretation is that the wave-length must be assumed small compared to the linear dimensions of the reflecting surface. If wave-lengths are given correctly by de Broglie's formula they are, as has been mentioned, of the same order as those of X-rays. This explains why reflections such as exhibited in Fig. 1 are obtained from the face of a crystal, but not from a polycrystalline surface, however highly polished; the reflected beam is synthesized, so to speak from a multitude of scattered waves spreading out from atoms regularly arranged in layers parallel to the surface—lacking this regularity the synthesis does not occur.

The specular reflection of X-rays from a crystal face is, as we know, selective in the following respect: the intensity of the reflected beam passes through sharp maxima as the glancing angle θ passes through

values which satisfy the Bragg formula $\sin \theta = n\lambda/2d$, where λ represents wave-length, d the distance between atom planes parallel to the surface, and n an integer. We expect this to occur with reflection of electrons, if the de Broglie waves are scattered by underlying atom layers as well as by the outermost. I will show later that the reflection portrayed in Fig. 1 is, indeed, selective. But with low-speed electrons, there are confusing complications which we can avoid by dealing with electrons of considerably higher speed; I will, therefore, begin by displaying the selective reflection of electrons accelerated through thousands rather than tens of volts only.

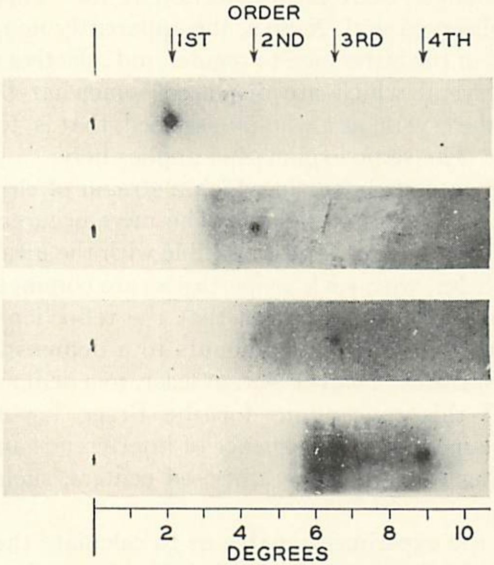


Fig. 2—Photographic record of selective reflection of 53-kv. electrons by a crystal of iron—(100) face.

In Fig. 2 we have a photographic record of the reflection of a beam of 53 kilovolt electrons from a crystal of iron. The experimental arrangement is essentially the same as that indicated in Fig. 1, though devices rather more elaborate are required for producing electron beams of such speeds. A photographic plate set at right angles to the direction of the primary beam and 30 or 40 cms. beyond the crystal replaces the exploring electrode. Each strip in Fig. 2 is the record of reflection at a particular angle of glancing. After each exposure the crystal is turned through the position of grazing (zero glancing angle), the primary beam falls directly upon the plate, and its direction is thus recorded. These fiducial marks appear in a column on the left. The

glancing angles are proportional (since all of them are quite small) to the distances from the spots formed by the primary beam to the edges of the respective fogged regions; the fogging is produced by general or random scattering and its sharp cutoff on the left marks the intersection of the plane of the crystal with the photographic plate. If reflections were specular but non-selective, a spot due to the reflected beam would appear on each strip as far to the right of the fogging edge as the fiducial spot is to the left. This is not what is observed; there is strong specular reflection at a series of equally spaced angles, and at adjacent angles weaker reflection which apparently is not regular. In other ranges of angle, there is no reflection at all. This is exactly the phenomenon observed with X-rays, the apparently non-regular reflection is ascribed in the latter case to regular and selective reflection from parts of the crystal which are displaced somewhat from the mean orientation of the crystal as a whole—ascribed, that is, to imperfections in the crystal. The same explanation applies here.

Here then is an experiment in which a stream of electrons exhibits the properties of a beam of waves. The mere occurence of specular reflection is, as we have seen, incompatible with the idea that electrons are simple particles, with such properties as are commonly ascribed to particles. The further observation that the reflection is selective in accordance with the Bragg law amounts to a demonstration that we are dealing with trains of waves—or, at least, to a demonstration of the convenience of this conception—for the Bragg law is simply and accurately explained as a consequence of interference among scattered waves expanding from regularly disposed centers, such as the atoms of a crystal.

The data of the experiment enable us to calculate the length of the waves. From the Bragg law $\lambda = 2d \sin\theta/n$; the reflections are from the (100) atom planes of iron for which $d = 1.43 \times 10^{-8}$ cm. or 1.43 Ängstrom units; the value of $\sin\theta/n$ deduced from Fig. 2 and related data is 0.0189, so that $\lambda = 2 \times 1.43 \times 0.0189 = 0.054$ Ängstrom units. *This is the experimentally determined wave-length of 53 kv. electrons.*

We compare this with the *theoretical* wave-length computed from the de Broglie formula, $\lambda = h/p$. The momentum p of a particle of rest-mass m and charge e which has been accelerated from rest through a potential difference V in absolute units is given in relativistic mechanics by

$$p = (2Vem)^{1/2}\left[1 + \frac{Ve}{2mc^2}\right]^{1/2},$$

where c represents the velocity of light. Writing this into the de Broglie formula and evaluating constants for the electron one obtains as a close approximation.

$$\lambda = \left(\frac{150}{V}\right)^{1/2} [1 - 4.9 \times 10^{-7} V]$$

for V in volts. Thus the theoretical wave-length of 150-volt electrons is one Ängstrom unit or 10^{-8} cm., and the wave-length of the 53 kv. electrons employed in this experiment is 0.0546 Ängstroms which is in good agreement with the value found experimentally.

These results which are from previously unpublished data by Davisson and Germer constitute perhaps the simplest demonstration of the wave aspect of electrons and verification of the de Broglie relation—the simplest, at any rate, in which use is made of crystal diffraction.

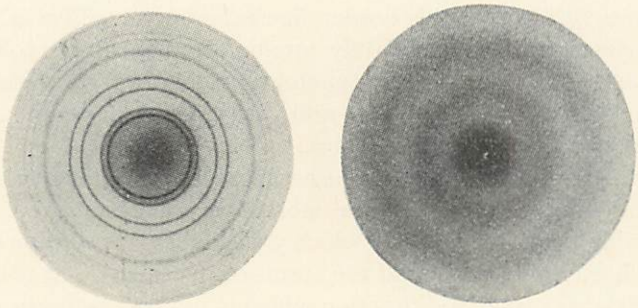
E. Rupp has demonstrated the diffraction of electrons by an ordinary ruled optical grating and has obtained thereby values of electron wave-lengths which agree with the theoretical values within the rather wide limits of error of this border line experiment. This is, in fact, an experiment more immediately intelligible than any involving the use of crystals. But Rupp's photographic plates exhibiting these results are not very impressive, and, therefore, I have not arranged for their reproduction in this report.

The Bragg reflection, though the easiest to interpret among the types of crystal diffraction, is not the most easily demonstrated, nor the most striking, nor yet the type which yields most information concerning the diffracting crystal and the atoms composing it. It is the Hull-Debye-Scherrer type of diffraction which is in these respects preeminent, and the type which has been most thoroughly investigated. A mass of finely divided crystals of random orientation is placed in the path of a beam of monochromatic radiation; a photographic plate set at right angles to the incident beam receives and records the radiation from the diffracting material. It is evident perhaps that the pattern produced by an aggregate of a very great number of crystals oriented at random will be the same as that generated by a single crystal turned into equally many random positions. Thus, if the material is iron, the single crystal in a particular orientation gives rise to the first order diffraction spot of Fig. 2; rotate the crystal about the primary beam and this spot generates a circle or ring—so also for the second and higher spots, each of them generates a ring.

But the atoms comprising the crystal may be regarded as arranged in many different sets of planes—there are, in fact, an infinite number of such arrangements. Of these, one is unique in having a greater

spacing between planes than any other—a greater value of the constant *d*. The first order ring due to these planes is the smallest in the pattern; the first order rings due to other arrangements follow in the order of decreasing values of *d*. These first order rings plus their companions of higher orders constitute the complete pattern of the aggregate. From the sequence of ring diameters, one infers the arrangement of the atoms in the crystal; from the scale of the pattern, the ratio of crystal spacing or "constant" to radiation wave-length; from the relative intensities of the rings, the way in which the scattering power of the atom varies with angle. The ring pattern is thus a storehouse of information concerning both crystal and wave.

Patterns of this type for electrons were obtained first by G. P. Thomson; it was thus, in fact, that Thomson made his demonstration of electron waves. A great many such patterns have since been obtained and studied. Two beautiful examples by Wierl are reproduced in Fig. 3. The one on the left records the diffraction of a beam of



Ag $\qquad$ CCl_4

Fig. 3—On left—diffraction pattern produced by transmission of 45-kv. electrons through thin silver foil—by R. Wierl. On right—pattern obtained by transmission of 36-kv. electrons through CCl_4 vapor—by R. Wierl.

36 kv. electrons by a thin film of polycrystalline silver, the one on the right is for 45 kv. electrons diffracted by carbon tetrachloride vapor. The form of the pattern for silver—the particular sequence of ring diameters which it displays—is characteristic of the so-called face centered cubic arrangement of atoms, and agrees with the pattern similarly obtained with X-rays. The scale of the pattern with other data of the experiment, including the scale factor of the silver crystal, yields an "observed" wave-length of 36 kv. electrons in agreement with the value computed from the de Broglie formula.

The pattern for carbon tetrachloride is bracketed with that for silver because the chlorine atoms in this molecule have the same ar-

rangement as the atoms in the crystal of silver. Four atoms only are required to fix a simple crystal arrangement; the chlorine atoms in CCl_4 occupy the corners of a regular tetrahedron, and this figure determines the face centered cubic arrangement. The chlorine atoms, to which nearly the whole of the scattering is due, may thus be properly thought of as forming exceedingly small crystals of this structure; the vapor is thus a crystal aggregate similar to that of polycrystalline silver. The marked differences between the two patterns are due to the disparity in the number of atoms forming the individual crystals in the two cases. The number in the chlorine crystals—four each—is the smallest possible, and the "resolving power" of the crystal, regarded as an optical instrument, is in consequence the least possible. The form of the pattern for CCl_4 accords with the tetrahedral arrangement of the chlorine atoms; the scale of the pattern with the calculated wave-length of the electrons fixes the length of the tetrahedron edge.

The purpose of this report is accomplished with the description of these few representative experiments which reveal and demonstrate—so far as demonstration is possible—a wave aspect of electrons in conformity with de Broglie's conception. The experiments do not tell us in what medium, if any, the waves occur, with what speed they are propagated, whether they are longitudinal or transverse and capable of polarization—they tell us merely that when electrons reach an element of space from a given source simultaneously over different paths the resultant intensity (the number of electrons traversing the element per unit time, as we think of it) is not the arithmetic sum of various scalar contributions as we naturally expect it to be, but is given instead by the square of the sum of contributions which are vector quantities—in precisely the manner with which we are familiar in optics. We must make the addition as if we were dealing with superposed trains of waves—with due regard for phase as well as amplitude. This kind of addition is characteristic of trains of waves. When we find quantities which add together in this particular way we conclude that the quantities are, in fact, trains of waves; this is our reason for regarding light and X-rays as wave phenomena and it is our reason also for so regarding electrons.

Having demonstrated the convenience, if not indeed the necessity, of regarding electrons in certain circumstances as waves rather than as particles, we enquire naturally if these waves are refracted on passing from one medium to another like the waves of light and X-rays, and whether or not they are polarizable. We rather expect to find that they do exhibit refraction, for if the wave-length of a beam of electrons in vacuo is given by $\lambda = (150/V)^{1/2}$ we expect that on passing into a

metal of which the thermionic work function is φ the wave-length of the beam will be altered to $\lambda' = [150/(V + \varphi)]^{1/2}$. We expect, in other words, that the metal will have for electrons accelerated through V volts a refractive index given by $\mu = \lambda/\lambda' = (1 + \varphi/V)^{1/2}$. Something of this kind is indeed found experimentally, though the phenomenon is less simple than we have here imagined.

Evidence of refraction is contained in the experimental results displayed in Fig. 4. The ordinates of this curve are proportional to the

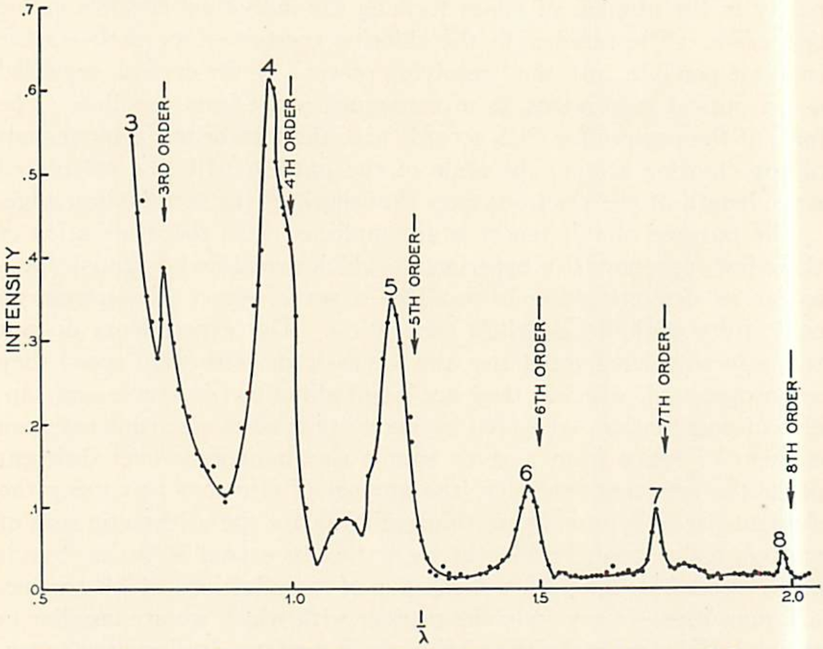


Fig. 4—Curve exhibiting selective reflection of electrons incident at 10 degrees on nickel crystal—(111) face. Departures from simple Bragg law reveal refraction.

intensity of the beam regularly reflected at 10 degrees incidence from a nickel crystal; the abscissa are proportional to the reciprocal of the wave-length of the incident beam. The maxima in the curve represent selective Bragg reflections of a sequence of orders, but are displaced somewhat to the left from the positions calculated from the simple Bragg formula, and indicated in the figure by arrows.

The simple Bragg formula is derived with the assumption, however, that the refractive index of the crystal is unity. The more general formula for reflection from a crystal face is $n\lambda = 2d(\mu^2 - \cos^2 \theta)^{1/2}$ which reduces to the familiar form when the index μ is equal to one. Thus if λ_1 represents the wave-length at which Bragg reflection of a

given order is expected ($\mu = 1$) and λ the wave-length at which the reflection maximum is actually observed, then the refractive index of the crystal satisfies the formula

$$(\mu^2 - 1) = \sin^2\theta\left(\frac{\lambda^2}{\lambda_1^2} - 1\right)$$

Or if in agreement with our assumed dispersion formula $\mu^2 - 1 = \varphi/V$ the constant φ will be given by $\varphi = (V_1 - V)\sin^2\theta$ where V_1 and V are the voltages corresponding to wave-lengths λ_1 and λ.

On our simple view of the matter, we expect this formula to yield the same value of φ for all orders of reflection and for all angles of incidence. This expectation is not, however, realized. From Fig. 4 we obtain for the value of φ for nickel 14 or 15 volts. But under other conditions values are obtained as low as 10 volts and as high as 20 or 25 volts. The results shown in Fig. 2, if rightly interpreted, also are incompatible with the assumed law of dispersion. Thus, if θ represents the calculated glancing angle at which the nth order Bragg reflection occurs for a given wave-length when $\mu = 1$ ($\varphi = 0$), then for $\varphi \neq 0$ this reflection is to be expected at angle θ_1 such that

$$\frac{\sin\theta}{\sin\theta_1} = \left(1 - \frac{4d^2\varphi}{150n^2}\right)^{1/2}.$$

It will be noted that the right hand side of the equation does not involve the speed or wave-length of the incident electrons from which we conclude that relative defections from the simple Bragg law should be as conspicuous for high speed electrons as for low, and this we find not to be the case; the reflections recorded in Fig. 2 conform to the simple law, as if φ were equal to zero. The situation then is this, that while we have clear evidence that electron waves are refracted, the laws of their refraction are evidently not simple, and are yet to be discovered.

I turn now to the polarization of electron beams. Rupp, in a recent series of remarkable experiments, has shown that a beam of electrons may, in appropriate circumstances, exhibit an asymmetry with respect to its direction of motion. In these experiments, beams of electrons are diffracted by thin films of gold and annular patterns are obtained like the one for silver shown in Fig. 3. They differ, however, from the patterns ordinarily obtained in that the individual rings are not uniformly dark all around, or as we say, in all "azimuths." In a certain azimuth the density of each ring is at a maximum, in the opposite azimuth (180° away) it is at a minimum—the rings are stronger or denser on one side of the pattern than on the other. This signifies a

non-uniformity or asymmetry in the number of electrons scattered into the various segments of a ring, and is due to an asymmetry in the source of the electrons constituting the beam incident upon the gold foil. These, instead of coming directly from a filament or from a high voltage discharge, as is ordinarily the case, are electrons which have already been scattered through 90 degrees by a metal target. The primary beam of electrons moved in a line parallel to the diffracting film and at some distance away; it fell upon a metal target; some of its electrons were scattered through 90° by this target, forming a secondary beam which was the one diffracted by the film of gold. Azimuths about a secondary beam formed in this way are not indistinguishable: one of them is unique in containing the primary beam which fell upon the target. It is in this azimuth that the electrons are most copiously scattered by the diffracting film, and the rings are most dense.

The sense of the effect may be stated in a different way. If the density of the rings were independent of azimuth, the mean total deflection from the primary beam (the deflection at the target plus the deflection at the film) of the electrons forming the pattern would be 90°: but actually, the mean deflection is greater than 90°. The electrons in the secondary beam have a bias toward a further deflection in the same sense as that which they received at the target.

Rupp has further shown that the effect of passing the secondary beam (the beam from the target to the film) through a *longitudinal* magnetic field is, so to speak, to rotate the azimuth of polarization; the azimuth of maximum density is displaced from its original position (without magnetic field) in a sense which depends upon the direction of the field and by an amount proportional to the length and intensity of the field.

The effect of a *transverse* magnetic field is different for different azimuths; if the direction of the field is normal, to the plane of the primary beam incident upon the target and the secondary beam the effect is nil. If it is parallel to this plane the polarization of the secondary beam decreases as the field strength is increased; at a critical intensity depolarization is complete, and at intensities which are higher still, the polarization is reversed—the azimuths of maximum and minimum density are interchanged. In these tests with transverse magnetic fields the force on the electron due to its motion through the magnetic field is balanced by the force due to a suitably adjusted electrostatic field.

The results obtained by Rupp in these experiments may be explained by means of the concepts of electron spin and corresponding

magnetic moment. The spin-axes of the electrons of the primary beam we imagine to be oriented at random. But among the electrons scattered by the target this is no longer true; those scattered in any small solid angle have a non-uniform distribution in orientation; the resultant of all their spins is a vector normal to the plane containing the primary beam and the direction of scattering. Thus, the resultant spin vector is everywhere tangent to circles about the primary beam, in the same general way as the magnetic vector in the field about a wire carrying an electric current. If the polarization process is one of selection—if, that is, the scattering material selects from the incident electrons those of a particular orientation to scatter in a particular direction, then it is to be expected that double scattering will exhibit just the type of asymmetry which Rupp actually observes.

The effect of a magnetic field upon a spinning electron is to cause its spin vector to precess about an axis parallel to the direction of the field. The same statement may be made in regard to the resultant spin vector of an assemblage of electrons such as that which constitutes the beam incident upon the gold foil in Rupp's experiments. This action is competent to explain the various changes which Rupp observes in the state of polarization of the primary beam when it has traversed the differently directed fields. The magnitude of the effect yields a value of the ratio of the magnetic moment of the electron to its angular momentum and this agrees well with the theoretical value of this ratio, e/mc.

It will have been remarked perhaps that in picturing these polarization effects, we have reverted to the conception that electrons are particles. It is difficult to see how they are to be explained in terms of the characteristics of waves. There is, however, no particular reason for making the effort. Here, as in other circumstances, the choice of conception is entirely a matter of convenience.

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